MEXT Scholarship

Math for undergraduate applicants

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MEXT Scholarship
Undergrad Math

Your math eBook guide to applying for
the Monbukagakusho Undergraduate Scholarship in Japan

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$$f(x) = x^2 - ax - a + 3$$
$$\Delta_x = (-a)^2 - 4(-a + 3) = a^2 + 4a - 12$$
$$\Delta_x = 0 \Longrightarrow a^2 + 4a - 12 = 0$$

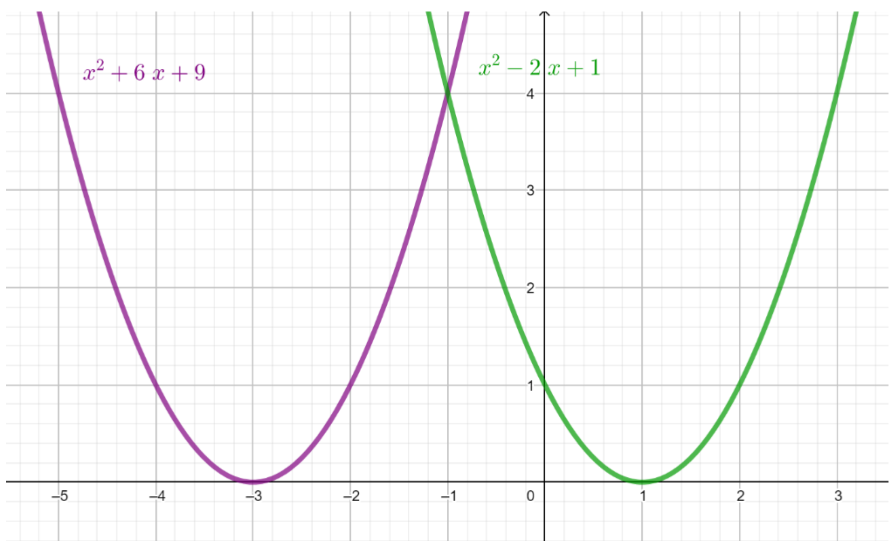
We will note the function like this: $g(a) = a^2 + 4a - 12$ and next, we have to find the roots.

$$\Delta_a = 4^2 - 4 * (-12) = 16 + 48 = 64$$
$$a_{1;2} = \frac{-4 \pm \sqrt{\Delta_a}}{2} = \frac{-4 \pm 8}{2} = -2 \pm 4$$
$$a_1 = -6$$
$$a_2 = 2$$

When "a" takes these two values, then the initial function looks like this:

$$a = -6 \Longrightarrow f(x) = x^2 + 6x + 9$$
$$a = 2 \Longrightarrow f(x) = x^2 - 2x + 1$$

And of course, these two functions are tangent to the x −axis. They intersect the axis in exactly one point as we can see in the graphs below.



Final answers: $-6; 2$

So, we found out the angle at which the function reaches its maximum, however, to actually find out the maximum value, we need to compute $f(150°)$.

$$f(150°) = \sin 150° - \sqrt{3}\cos 150°$$

We will use the following formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

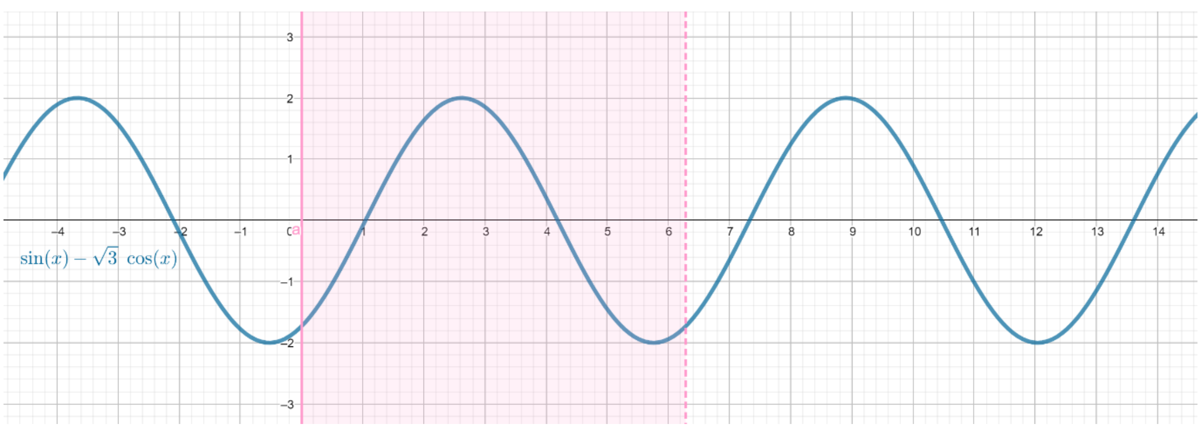
$$\sin 150° = \sin(90° + 60°) = \sin 90° \cos 60° + \sin 60° \cos 90° = 1 * \frac{1}{2} + \frac{\sqrt{3}}{2} * 0 = \frac{1}{2}$$

$$\cos 150° = \cos(90° + 60°) = \cos 90° \cos 60° - \sin 90° \sin 60° = 0 * \frac{1}{2} - 1 * \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$f(150°) = \sin 150° - \sqrt{3}\cos 150° = \frac{1}{2} - \sqrt{3} * \left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

$$f(150°) = 2$$

This means that the maximum value for the function is 2, and it can be reached when the angle $\theta = 150°$. We can see below how the graph of the function looks like and also that the maximum value is 2. I marked with pink the domain for our exercise $(0 \le \theta < 360°)$.



Final answer: 2

1.5 If $\boldsymbol{x + y = 3}$ and $\boldsymbol{x^2 + y^2 = 5}$, then $\boldsymbol{x^3 + y^3} = \square$.

Solution:

$x + y = 3$; we square both sides

$x^2 + y^2 + 2xy = 9$

$2xy = 9 - 5$

$2xy = 4$

We can see from the graph that the lines are perpendicular.

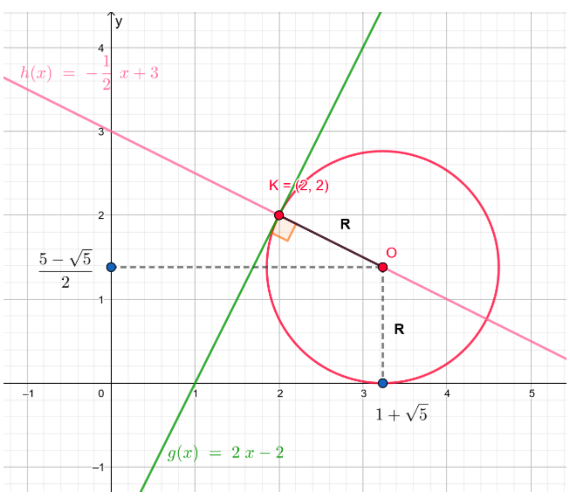
In our problem we have:

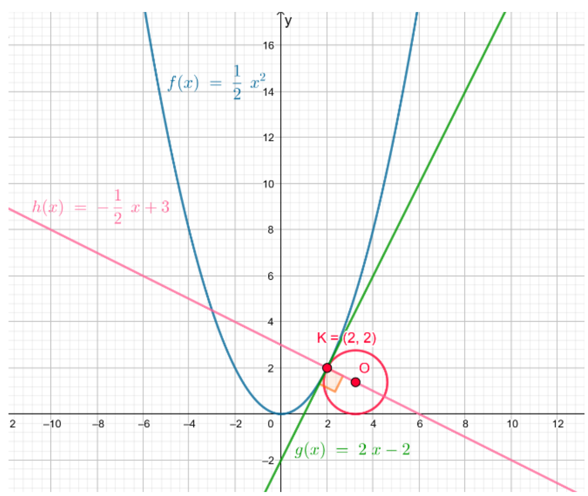
$$h(x) \perp g(x) \iff c * 2 = -1$$
$$c = -\frac{1}{2}$$

$$h(x) = -\frac{1}{2}x + d$$
$$h(2) = -\frac{1}{2} * 2 + d = -1 + d = 2$$
$$d = 3$$

The equation of the line which passes through KO:

$$h(x) = -\frac{1}{2}x + 3$$





(3) Next, we need to find out the coordinates of the circle center.

Circle equation:

$$(x - a)^2 + (y - b)^2 = R^2$$

This means that the center 'O' has the coordinates a and b.

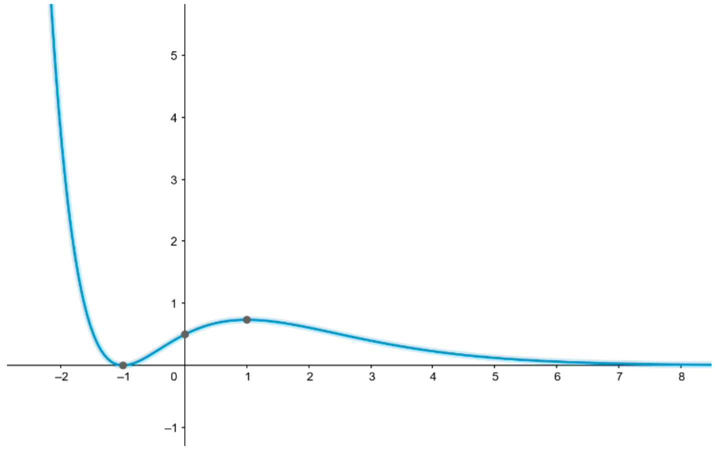
We will note: $O(a, b)$.

problem asks us to find the maximum of this function. So, the question is, when is the triangle maximum and what is the value.

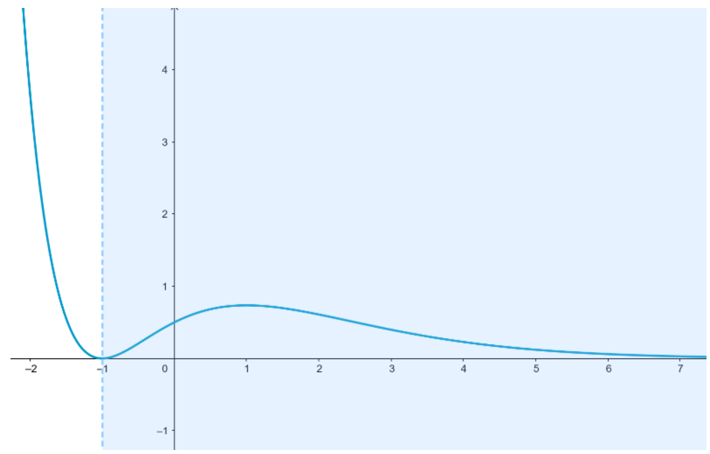
$S(a) = \frac{1}{2}e^{-a}(a+1)^2$

Find the maximum of S_a

This is how the function looks like.



Let's not forget that $a > -1$



Now, we need to find the extreme points. To do that, we will derivate it. We'll apply the following rules:

$(f * g)' = f' * g + f * g'$

$(e^u)' = e^u * u'$

$(u^2)' = 2u * u'$

$$S'(a) = \frac{1}{2}[(e^{-a})' * (a+1)^2 + (e^{-a}) * [(a+1)^2]']$$

$$(e^{-a})' = e^{-a} * (-a)' = e^{-a} * (-1) = -e^{-a}$$

$$[(a+1)^2]' = 2a + 2$$

$$(a^2 + 2a + 1)' = 2a + 2$$

$$S'(a) = \frac{1}{2}[-e^{-a}(a+1)^2 + e^{-a}(2a+2)] = \frac{1}{2}[e^{-a}(-(a+1)^2 + 2a + 2)]$$

$$= \frac{1}{2}e^{-a}(1 - a^2) = \frac{1}{2}e^{-a}\,(1-a)(1+a)$$

We know from the exponential function conditions that it cannot be negative or zero. So, $\frac{1}{2}e^{-a} > 0$

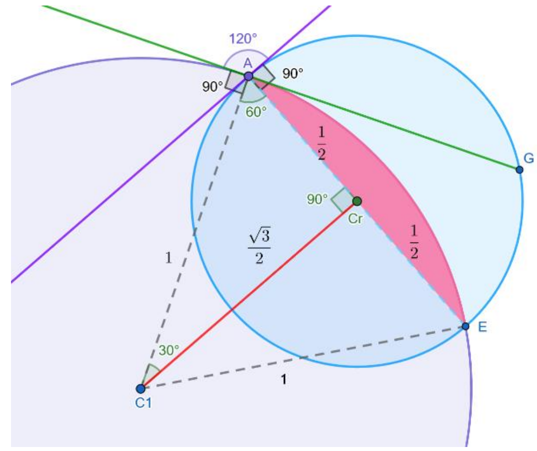
$$S'(a) = 0$$

$$\frac{1}{2}e^{-a}\,(1-a)(1+a) = 0 \Rightarrow a = 1 \text{ or } a = -1$$

However, the condition of the problem is that $a > -1$, which means that $a = 1$

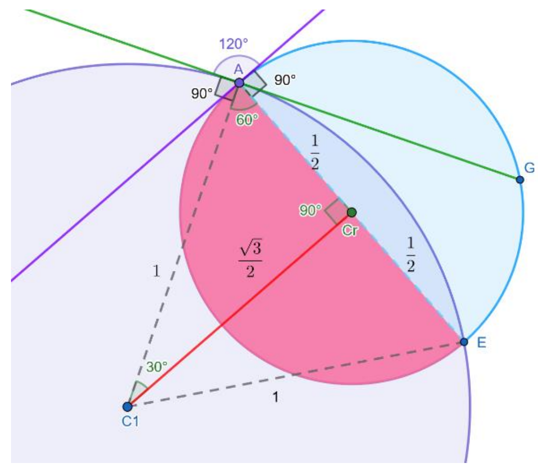
Area of the segment $\widehat{AE}$ = circle sector $AC_1E - S_{\Delta AC_1E}$

$$S_{\widehat{AE}} = \frac{\pi}{6} - \frac{\sqrt{3}}{4}$$



AC_rE is a diameter in the circle C_r. Therefore, AE divides the circle into two segments of equal areas. The area of $\widehat{AE}$ belonging to the blue circle (pink area) has the following area:

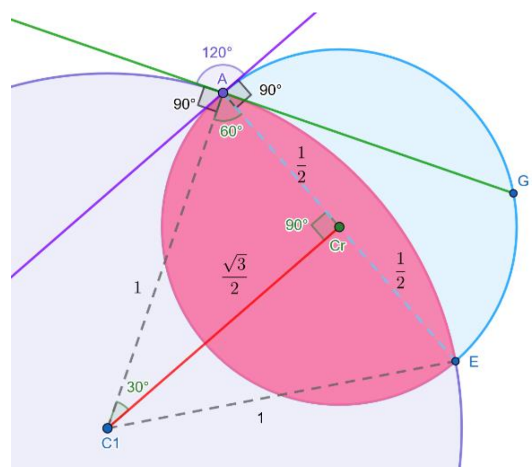
$$\frac{1}{2} * \pi r^2 = \frac{1}{2}\pi * \frac{1}{4} = \frac{\pi}{8}$$



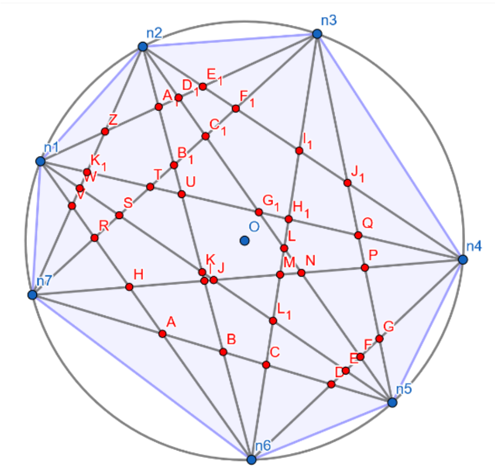
Now, all we have to do is add the two areas, and we get the answer to the question:

$$\frac{\pi}{6} - \frac{\sqrt{3}}{4} + \frac{\pi}{8} = \frac{7\pi}{24} - \frac{\sqrt{3}}{4}$$

Final answer: $\frac{7\pi}{24} - \frac{\sqrt{3}}{4}$



We should calculate the intersections for $n = 7$ and $n = 8$ as well, because the data is not enough to find a pattern here.



For $n = 7$, there are 35 intersections. Also, there are 57 regions.

$$i_7 = 35$$

$$r_7 = 57$$

We can also apply the formula to see the number of chords, and then count them just to make sure the formula is correct.

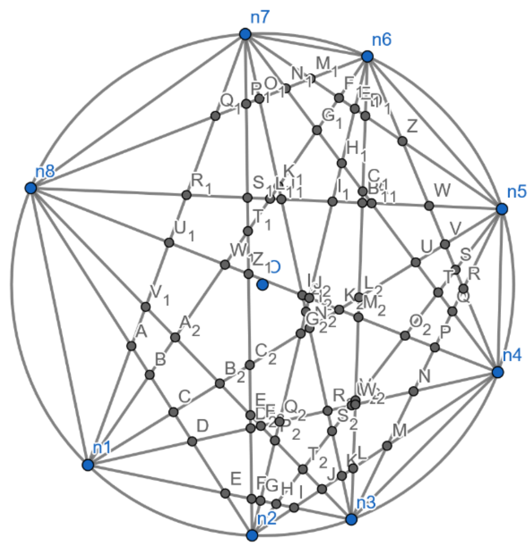
$$C_n^2 = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)}{2}$$

$$C_7^2 = \frac{7!}{2! * 5!} = \frac{6 * 7}{2} = 3 * 7 = 21$$

So, for $n = 7$, we have 21 chords.

For $n = 8$, there are 70 intersections.

$$i_8 = 70$$



PART IV - TABLES