# MEXT Scholarship

## Math for Special Training College

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# MEXT Scholarship
# Special Training College Math

*Your math eBook guide to applying for the Monbukagakusho*

*Special Training College Scholarship in Japan*

My Japanese Experience Series

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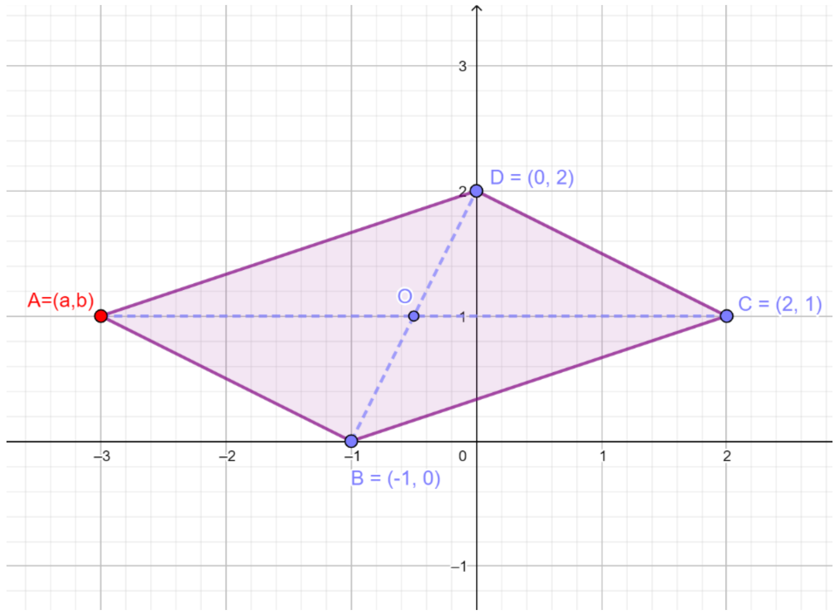
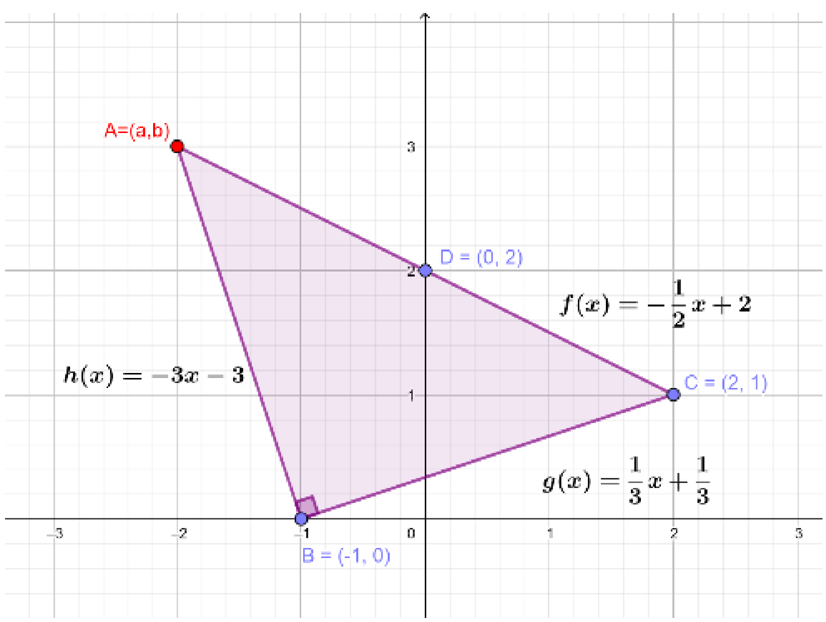
$$A = (-3; 1)$$




(3) For the next part, the angle $\sphericalangle ABC = 90°$ and $D \in AC$. To find the coordinates of $A(a, b)$, we can approach the problem in two ways. Either we use the line function and slope all the way, or we have a mix of line function, slope and the Pythagorean theorem. I will do both below.

$(I)$ We will first start with the line functions.




We will note the line for $DC$ with $f(x) = jx + k$ and then we use the coordinates of $D(0; 2)$ and $C(2; 1)$ to find out the function.

Solution:

(*i*) $f'(x) = 3x^2 - 6 * 2x + 9 = 3x^2 - 12x + 9$

Find the roots of $3x^2 - 12x + 9 = 0$. Divide by $3$ and we obtain:

$$x^2 - 4x + 3 = 0$$

$$\Delta = 4^2 - 4 * 3 = 16 - 12 = 4$$

$$x_{1;2} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} = 2 \pm 1 \Longrightarrow \begin{cases} x_1 = 1 \\ x_2 = 3 \end{cases}$$

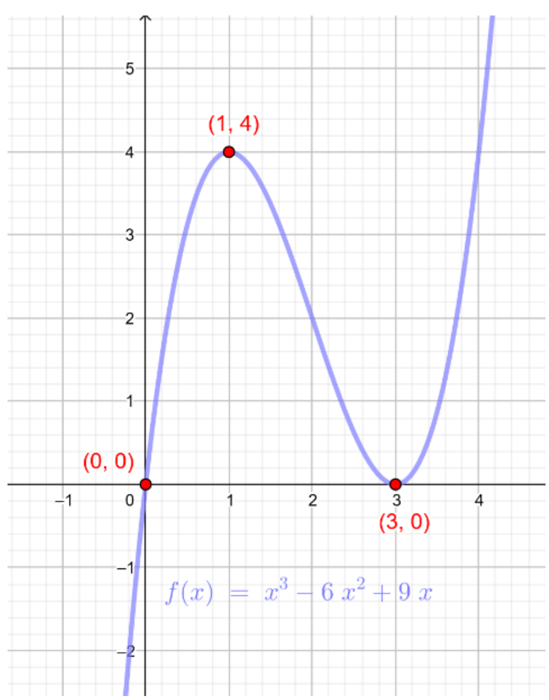
$$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$$

To find out the $x$ for which the function has a maximum value, we need to make the table of variation. First, we find out the roots of the function.

$$f(x) = x^3 - 6x^2 + 9x = x(x^2 - 6x + 9) = x(x-3)^2$$

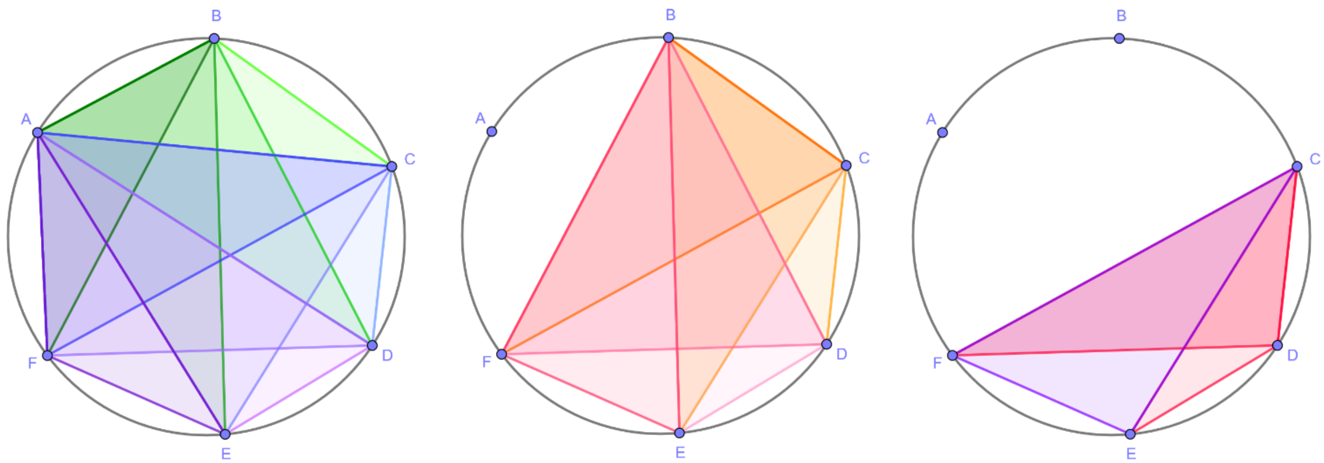
| $x$ | | 0 | | 1 | | 3 | |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | | | + + + + + + + + + | 0 | − − − − − | 0 | + + + + + |
| $f(x)$ | ↗ | 0 | ↗ | | ↘ | 0 | ↗ |

As we can see from the table, the function increases until $1$ and then it decreases. This means that here we have a maximum, where $x = 1$ and $f(1) = 4$. Then the function decreases until $x = 3$, and then it increases again. This means that in $x = 3$ we have a minimum. You can see below how the graph looks like:

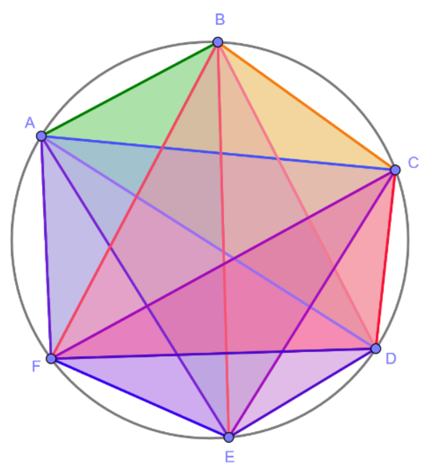


In total we have:

$$10 + 6 + 4 = 20$$




In the next picture are all $20$ triangles:




Final answer: 20

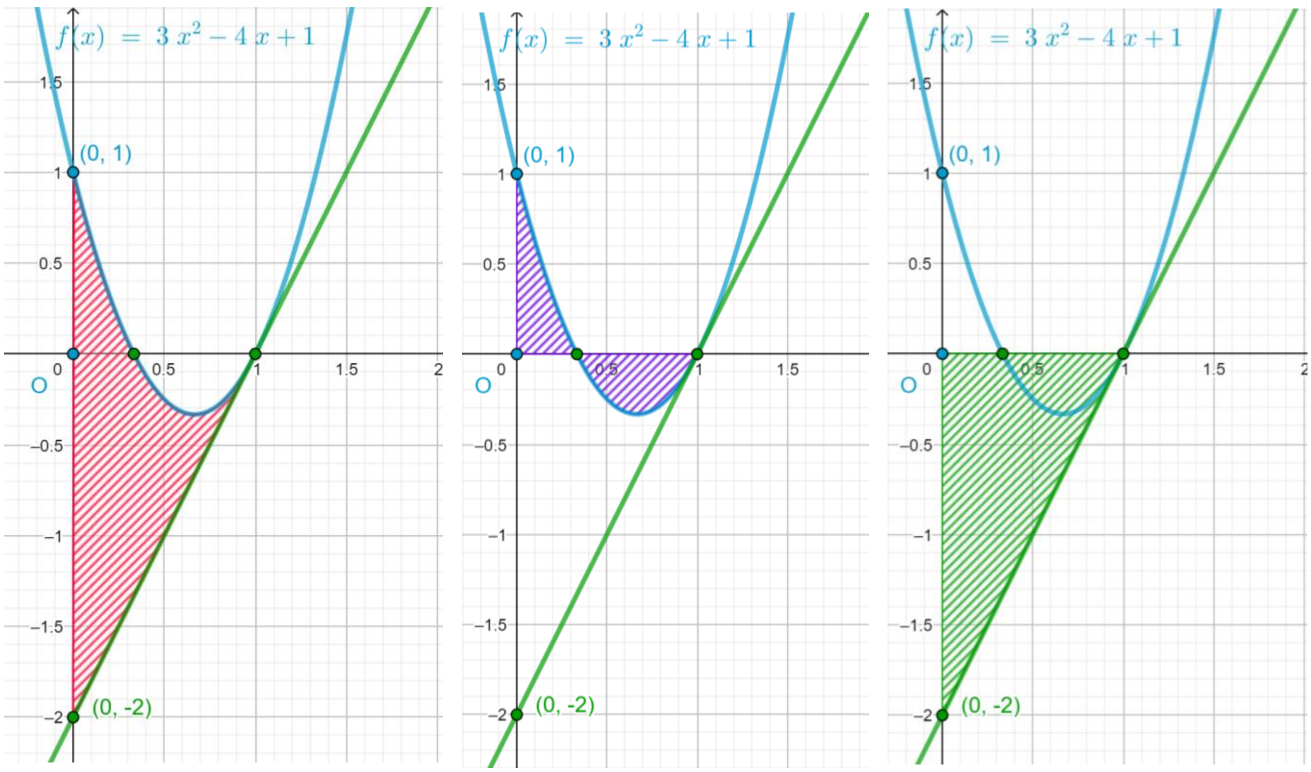
$(II)$

The second way of solving this is by using combinations. In how many ways can we combine the points to obtain triangles. We need three points for each triangle, so we will use a combination of $6$ taken $3$ at a time:

$$C_6^3 = \frac{6!}{3!\,(6-3)!} = \frac{4*5*6}{1*2*3} = 4*5 = 20$$

This is a much faster way of finding out the number of triangles, and also, we can answer more complex questions such as ‘How many triangles can we draw if we have $100$ points on the circle?’ And the answer is:

Next, we need to calculate the area between $f(x)$, $g(x)$, and $Oy$, as we can see on the first graphic, drawn with red. Part of the area is above $Ox$ and is positive, and part of the area is below $Ox$ and is negative. Therefore, we have to compute each part separately. Typically, we would compute the area of the green triangle as seen on the third graphic, then subtract the area between the roots and add the area above the $Ox$ −axis. However, in this case, is easier as we will see below because the two purple areas we can see on the middle graphic (above $Ox$ and between the roots) are actually equal. This means that the red area will be equal to the green area.

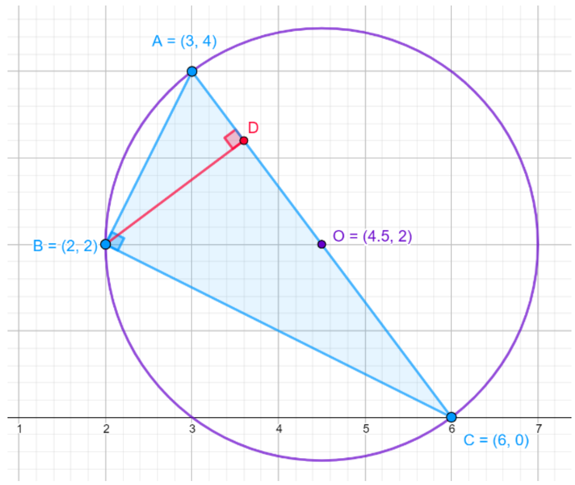



The area above $Ox$:

$$\int_0^{\frac{1}{3}} f(x)\,dx = \int_0^{\frac{1}{3}} (3x^2 - 4x + 1)\,dx = (x^3 - 2x^2 + x)\Bigg|_0^{\frac{1}{3}} = \frac{1}{3^3} - 2 * \frac{1}{3^2} + \frac{1}{3}$$

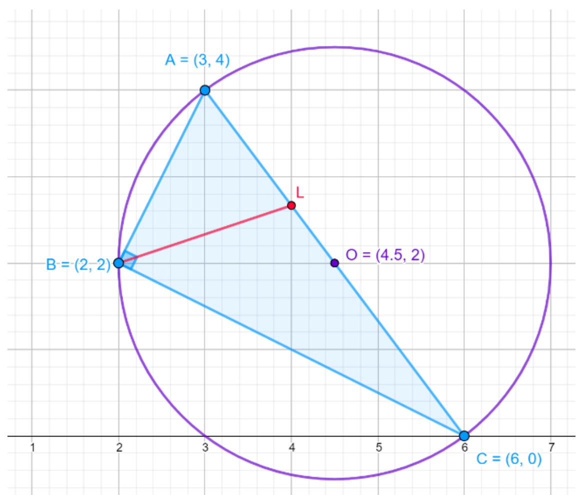
$$= \frac{1}{27} - \frac{6}{27} + \frac{9}{27} = \frac{4}{27}$$

The area between the roots can be computed in two ways:




$$BD = \frac{AB * BC}{AC} = \frac{\sqrt{5} * 2\sqrt{5}}{5} = 2$$

(5) When point $L$ is the intersection of the bisector of $\sphericalangle ABC$ and side $AC$, then $AL: LC = 1:\square$.




We draw the bisector $BL$, and then we apply the bisector theorem:

$$\frac{AL}{LC} = \frac{AB}{BC}$$

$AB = \sqrt{5}$; $BC = 2\sqrt{5}$

$$\frac{AB}{BC} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2} \Longrightarrow \frac{AL}{LC} = \frac{1}{2}$$

(6) $\overrightarrow{BL} = \square\,\overrightarrow{BA} + \square\,\overrightarrow{BC}$.