MEXT Math A Scholarship

- nine years of tests
- step-by-step solutions
- beginner and intermediate level
- undergrad students

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MEXT Scholarship
Undergrad Math A

Your math eBook guide to applying for
the Monbukagakusho Undergraduate Scholarship in Japan

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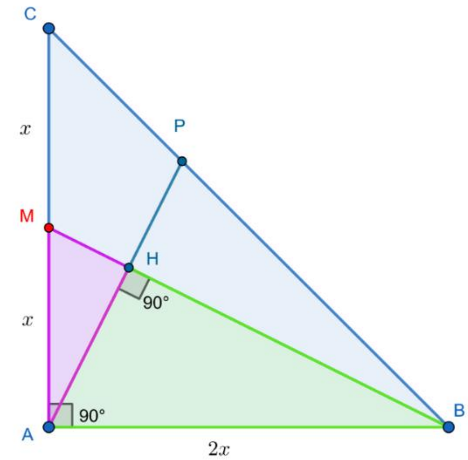
I will mark them with green and purple so we can see them better.



Find out BM within ΔABM, which is a right triangle, $\measuredangle A = 90°$.

We apply the Pythagorean theorem:

$$BM^2 = AB^2 + AM^2 = (2x)^2 + x^2 = 4x^2 + x^2 = 5x^2$$

$$BM = \sqrt{5x^2} = x\sqrt{5}$$

$AH \perp BM$; so, we apply the altitude theorem:

$$AH = \frac{x * 2x}{x\sqrt{5}} = \frac{2x}{\sqrt{5}} = \frac{2\sqrt{5}x}{5}$$

ΔABH is a right triangle; $\measuredangle AHB = 90°$

We apply the Pythagorean theorem to find out BH:

$$AB^2 = AH^2 + BH^2$$

$$BH^2 = AB^2 - AH^2 = (2x)^2 - \left(\frac{2\sqrt{5}x}{5}\right)^2 = 4x^2 - \frac{20x^2}{25} = 4x^2 - \frac{4x^2}{5} = \frac{20x^2 - 4x^2}{5} = \frac{16x^2}{5}$$

$$BH^2 = \frac{16x^2}{5} \Longrightarrow BH = \sqrt{\frac{16x^2}{5}} = \frac{4x}{\sqrt{5}} = \frac{4\sqrt{5}x}{5}$$

$$BH = \frac{4\sqrt{5}x}{5}$$

Next, we need to find out MH.

We can either apply the Pythagorean theorem in ΔAHM or the altitude theorem (I) in ΔAMB. The results have to match. I'll do them both below.

ΔAHM - Pythagorean theorem

$$AM^2 = MH^2 * AH^2$$

We will show that $\tan 60° = \sqrt{3}$:

$$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Now, we will rewrite the function using the tangent.

$$f(\theta) = \sin\theta - \tan 60° \cos\theta = \sin\theta - \frac{\sin 60°}{\cos 60°}\cos\theta = \frac{\cos 60° \sin\theta - \sin 60° \cos\theta}{\cos 60°}$$

We'll use the following formula:

$$\sin(a - b) = \sin a \cos b - \sin b \cos a$$

$$f(\theta) = \frac{\sin(\theta - 60°)}{\cos 60°} = \frac{\sin(\theta - 60°)}{\frac{1}{2}} = 2\sin(\theta - 60°)$$

We rewrite the function $f(\theta)$ as following:

$$f(\theta) = 2\sin(\theta - 60°)$$

"2" is a constant and it cannot be maximized. To maximize the function, we need $\sin(\theta - 60°)$ to be as large as possible within our initial condition: $(0 \leq \theta < 360°)$

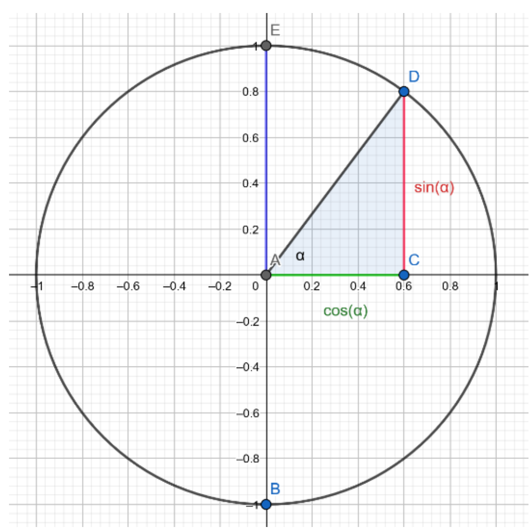
We also have another condition, it is not given in the problem, but is the existential condition of the sine function:

$$\sin x \in [-1; 1]$$

This means that the maximum of the sine function is 1. It cannot be greater than that. This means that for $\sin(\theta - 60°)$ to be maximum, it has to be equal to 1:

$$\sin(\theta - 60°) = 1$$

The sine function is 1 when the angle is $90°$.



As we can see on the trigonometric circle or the unit circle, DC (the red line) is the sine, and AC (the green line) is the cosine. When $\alpha = 90°$, the sine (AE – blue line) is equal with 1.

$\sin 90° = 1$

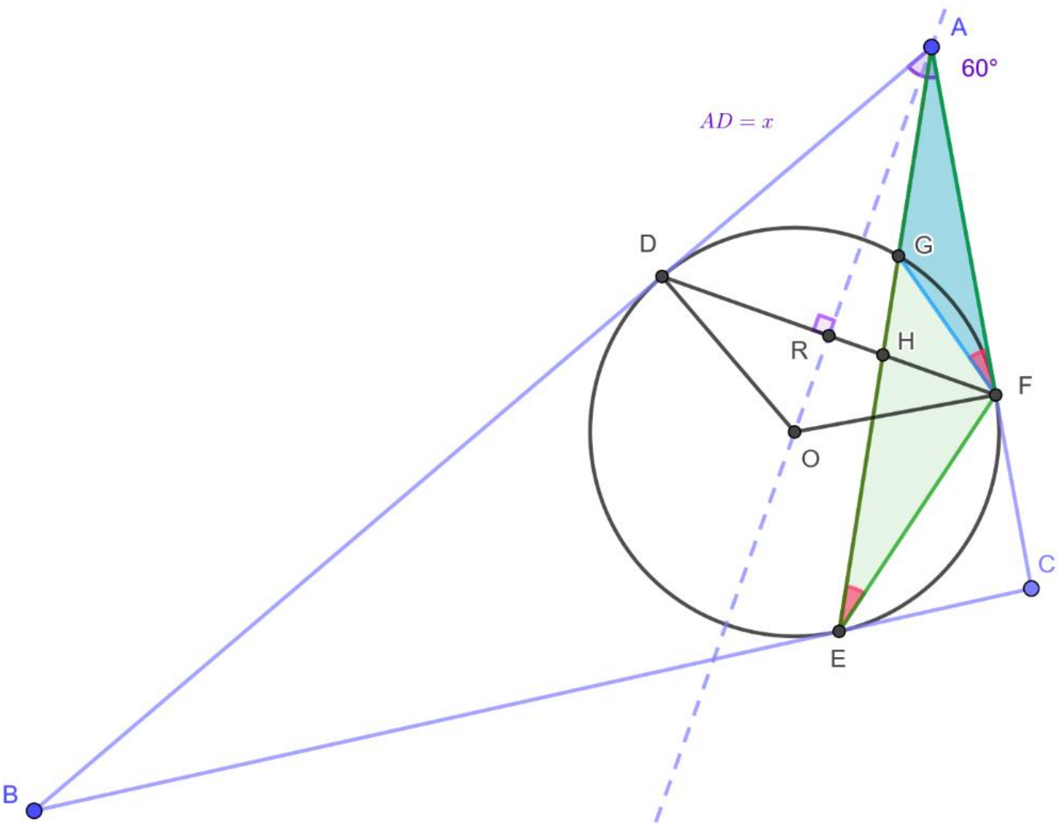
If we return to our problem, $\sin(\theta - 60°) = 1$, which means

$\theta - 60° = 90°$

$\theta = 90° + 60°$

$\theta = 150°$

$$\frac{AG}{AF} = \frac{AF}{AE} \Longrightarrow AG * AE = AF^2$$



We already know that $AF = x$, and this would mean that $AG * AE = x^2$.

So far, it is guesswork. Now, we must prove this and make sure that it is true.

We want to prove the similarity of the triangles ΔAGF and ΔAFE.

$$\Delta AGF \sim \Delta AFE$$

For this to be true, the angles must be equal:

$$\begin{cases} \sphericalangle AGF = \sphericalangle AFE \\ \sphericalangle GAF = \sphericalangle FAE \\ \sphericalangle AFG = \sphericalangle AEF \end{cases}$$

$\sphericalangle GAF = \sphericalangle FAE$ this is true because it is actually the same angle.

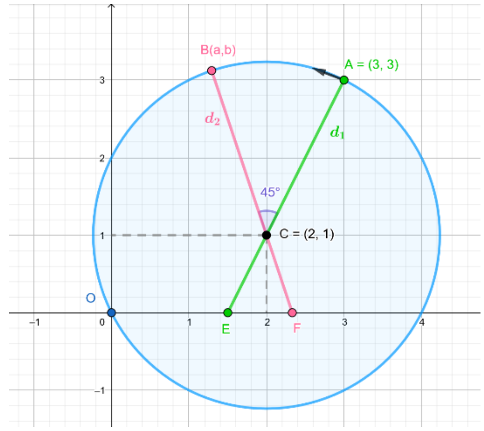
We will note $\sphericalangle AEF = r$.

$\sphericalangle AEF$ has the vertex on the circle, so the intercepted arc $\widehat{GF} = 2r$.

$\sphericalangle DAF = 60° \Longrightarrow \sphericalangle DOF = 180° - 60° = 120°$

First way:

The first line will be d_1 (green), the second line will be d_2 (pink).



$$d_1: \quad C(2,1) \in d_1; A(3,3) \in d_1$$
$$d_2: \quad C(2,1) \in d_2; B(a,b) \in d_2$$

We need to find $B(a,b)$.

First, let's find out the equation of the line d_1

d_1: $\quad y = px + t$

$C(2,1)$: $\quad 1 = p * 2 + t$

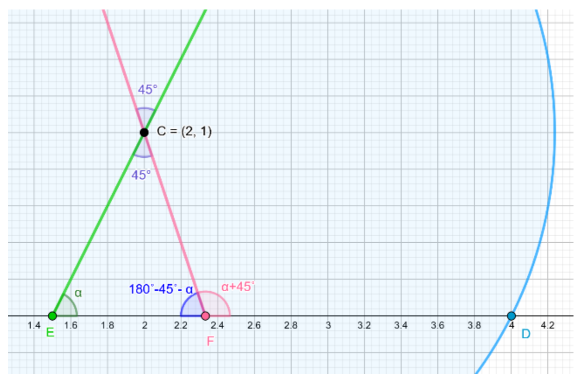
$A(3,3)$: $\quad 3 = p * 3 + t$

$$\begin{cases} 2p + t = 1 \\ 3p + t = 3 \end{cases} \Longrightarrow \begin{cases} t = 1 - 2p \\ 3p + 1 - 2p = 3 \end{cases} \Longrightarrow \begin{cases} p = 2 \\ t = -3 \end{cases}$$

The equation of d_1 is $y = 2x - 3$

Next, we want to find the equation of the line d_2, and for this, we will need its slope.

We know that the angle of rotation is 45°



$\sphericalangle ACB = 45° \Longrightarrow \sphericalangle ECF = 45°$

$\sphericalangle CEF = \alpha \Longrightarrow \sphericalangle EFC = 180° - 45° - \alpha$

$\sphericalangle EFD = 180° \Longrightarrow \sphericalangle CFD = 180° - \sphericalangle EFC$

$\sphericalangle CFD = 180° - (180° - 45° - \alpha)$

$\sphericalangle CFD = 45° + \alpha$

Basically, we take all numbers from Question B, we subtract the mean (which is 5.0), and then we square each result. After that, we add the results, divide everything by the number of the sample (in our case 10), and then extract the square root.

$$S_N = \sqrt{\frac{(-2)^2 + 0 + 1^2 + (-1)^2 + 0 + 3^2 + 1^2 + 0 + 0 + (-2)^2}{10}}$$

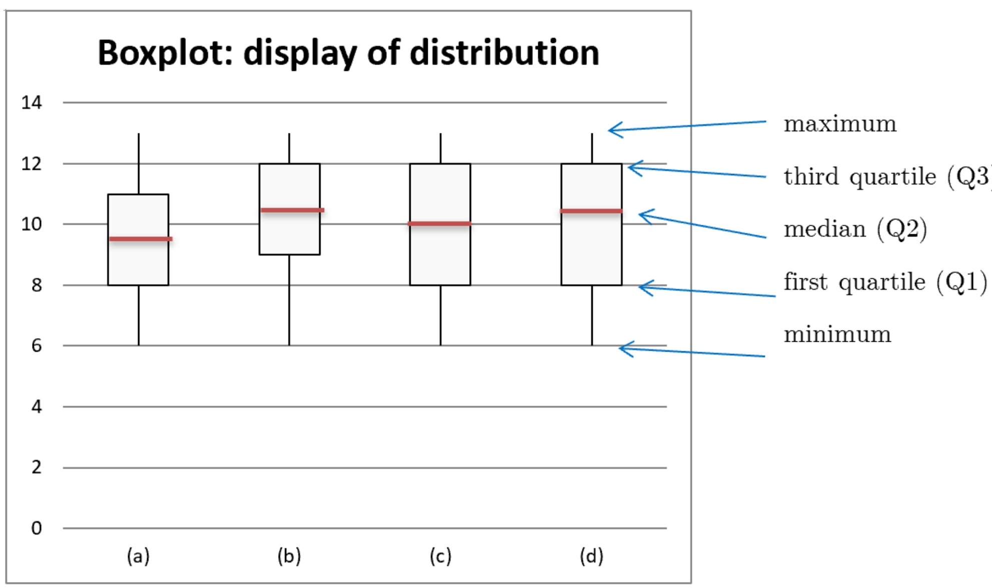
$$= \sqrt{\frac{4 + 1 + 1 + 9 + 1 + 4}{10}} = \sqrt{\frac{20}{10}} = \sqrt{2} = 1.4$$

The standard deviation for Question B is 1.4. The answer to $\boxed{[3-5]}$ is 1.4.

The standard deviation for Question A: $S_N = \sqrt{\frac{44}{10}} = \sqrt{4.4} = 2.09 \sim 2.1$

The standard deviation for the Total: $S_N = \sqrt{\frac{64}{10}} = \sqrt{6.4} = 2.5$

(2) Next, we will solve the mystery of the boxplot.



Minimum – refers to the minimum number from the data set.

Q1 (first quartile) – is the middle number between the smallest number and the median of the dataset.

Q2 (median) – the median is the "middle value." To see which one it is, we have to order all numbers from the data set, from the smallest to the largest, and take the number in the middle. If there are two numbers in the middle, we calculate the average.

Example: